Appraising the "Merger Price" Appraisal Rule[*]

Albert H. Choi
University of Virginia Law School

Eric Talley
Columbia Law School

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Abstract

This paper develops an auction design framework to analyze various methods for assessing "fair value" in post-merger appraisal proceedings. Our inquiry spotlights an approach recently embraced by some courts benchmarking fair value against the merger price itself. We show that merger price deference effectively nullifies the role that appraisal can potentially play in establishing a *de facto* reserve price for company auctions, thereby depressing both acquisition prices and target shareholders' expected welfare relative to both the optimal appraisal policy and a variety of other valuation measures. We also examine conditions under which deference to the merger price can be optimal. Our results have empirical implications for understanding appraisal, and they likewise help to inform doctrine by providing guidance to legal actors about when a sales process can be considered sufficiently "robust" to justify merger price deference.

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Introduction

In mergers and acquisitions law, shareholders of a target company often enjoy a statutory right to reject the terms of an approved sale in favor of a judicial determination of "fair value" for their shares. All states provide dissenters with some form of appraisal right,[1] which—when triggered—provides a strategic tool for dissenters to resist deal terms they consider to be inadequate or under-compensatory. Historically uncommon, appraisal actions have grown increasingly prevalent in the last decade, a trend fueled by a combination of the 2008 Financial Crisis, several institutional reforms, and the emergence of several hedge funds employing appraisal as a quasi-arbitrage strategy (Jiang et al., 2016; Korsmo & Myers 2015; 2016; Callahan et al. 2018).

The renewed popularity of appraisal has also awakened longstanding jurisprudential uneasiness about the proceeding. Contrary to garden-variety lawsuits, appraisals do not assign a burden of proof explicitly to either side, and they require the presiding judge (acting without a jury) to deliver a single price per share at the end of the process (Talley 2017). Expert testimony—the chief source of evidence in such cases—usually clouds more than it clarifies, with opposing experts delivering valuation opinions that diverge substantially. Finally, appraisal invariably forces the factfinder to wander far into the underbrush of financial valuation techniques (such as discounted cash flow analysis) in order to divine fair value, a disquieting challenge for generalist judges. As one member of the Delaware Chancery Court put it:

> [T]his task is made particularly difficult for the bench judge, not simply because his training may not provide a background well-suited to the process, but also because of the way the statute is constructed…[I]n reality, the 'burden' falls on the judge to determine fair value, using all relevant factors. (In re Appraisal of Ancestry.com, 2015 WL 399726, at 2.)

In several recent appraisal opinions,[2] courts have begun to search for a convenient exit ramp from this computational conundrum. Delaware has led the way, growing increasingly willing to defer to *the merger price itself* as evidence (if not *the decisive* piece of evidence) of fair value. The (so-called) "Merger Price" (MP) rule began to make regular appearances in judicial decisions in 2010, and it has been a doctrinal mainstay ever since. The Delaware Supreme Court, moreover, has given its conditional endorsement of deal price deference, at least when the sales process appears adequately

[1] For public targets, Delaware law limits appraisal to statutory mergers involving cash consideration and squeeze-outs. See DGCL §262(b). Other states employ slightly different rules that are both narrower and broader than Delaware's. *See* RMBCA §§13.01-02.

[2] See, e.g., Merion Capital v. Lender Processing Services, C.A. No. 9320-VCL (Del. Ch. Dec. 16, 2016); Dunmire v. Farmers & Merchants Bancorp, C.A. No. 10589-CB (Del. Ch. 2016); Merion Capital v. BMC Software, C.A. No. 8900-VCG (2015); Huff Fund Inv. P'ship v. CKx, Inc., 2013 WL 5878807 (Del. Ch. Nov. 1, 2013); In re Appraisal of Ancestry.com, 2015 WL 399726 (Del. Ch. Jan. 30, 2015); LongPath Capital LLC v. Ramtron International Corp., C.A. No. 8094-VCP (Del. Ch. June 30, 2015); Merlin Partners LP v. AutoInfo Inc., C.A. No. 8509-VCN (Del. Ch. Apr. 30, 2015); The Union Illinois 1995 Investment Limited Partnership v. Union Financial Group, Ltd., 847 A.2d 340 (Del. Ch. 2004).

robust.[3] At present, however, case law remains unsettled as to what the conditions of robustness are, and how broad deal price deference should be once triggered.

This paper explores whether and when merger price deference is consistent with financial principles of value maximization. Building on a canonical auction-design model, we demonstrate that the prospective threat of post-merger appraisal can serve as a *de facto* "reserve price" in the auction that feeds back on sales process, inducing bidders to become more aggressive so as to reduce their subsequent exposure to appraisal risk. In serving this role, appraisal can enhance value whenever auction design principles would similarly prescribe the establishment of credible reserve price. As such, appraisal can work to benefit *all* shareholders—even those who are unlikely to dissent.

Under a rule of deal price deference, in contrast, the reserve-price benefits of appraisal immediately unravel. The MP rule dictates that appraised value mechanically floats up and down with the winning bid, regardless of the bid's adequacy. Opting for appraisal, therefore, can never yield a payoff exceeding merger price. Anticipating appraisal's functional irrelevance, rational buyers disregard appraisal risk when formulating strategy, softening their bids in the process. Viewed alone, merger price deference functionally vitiates the appraisal right, and whatever value enhancing reserve-price implications it might deliver. Absent other functional tools for establishing a value-enhancing reserve price, therefore, the MP rule is generally suboptimal.

That said, several institutional practices can provide serviceable substitutes for appraisal in establishing a credible reserve price, including: (1) bargaining tactics of the target company's board of directors; (2) the requirement of target shareholder approval; and (3) competitive auction procedures with several bidders. Our framework allows us to study how these factors interact, and we show that the right combination of them can indeed dampen the value-enhancing role of appraisal, thereby rehabilitating the case for deal price deference in certain circumstances. At the same time, the availability of such factors is far from universal. Target boards, for example, can suffer from agency costs and imperfect commitment, eroding their reliability as reserve-price sentries. Shareholder voting may similarly falter, particularly when the swing voter preferences are not representative of other shareholder interests. And eliciting bidder competition often requires both thoughtful design and significant effort—preconditions that are often not evident (and may be infeasible) in several company auctions. Consequently, even when one allows for other institutions establishing a reserve price, appraisal still can play an important, value-enhancing role for target shareholders.

We illustrate our claims using a formal auction design framework that explicitly incorporates managerial agency and commitment costs, shareholder voting, bidder competition, and post-merger appraisal rights. In our model, a target manager-agent runs an auction among a number of bidders. We reflect managerial agency cost by assuming

[3] DFC Global Corp. v. Muirfield Value Partners, 172 A.3d 346, at 348-9 (Del. 2017) (holding that the Chancery Court's valuation approach should be upheld when there is "reasonable basis [for it] in the record and in accepted financial principles relevant to determining [fair] value"). See also Dell Inc. v. Magnetar, 177 A.3d 1 (Del. 2017).

that the agent realizes a (positive or negative) private benefit in case the firm is sold. We also assume that the agent is unable to set a credible reserve price for the auction (beyond that which would satisfy its payoff *ex post*). After the winner of the auction emerges, the winning bid is subject to a vote by the target shareholders, who have differential preferences. Should a requisite fraction of shareholders vote in favor, the deal is consummated, but dissenting shareholders can seek judicial appraisal, with the court valuing shares by using pre-specified valuation criteria to determine "fair value."

We compare equilibria under "conventional" appraisal valuation approaches (where the valuation criteria are independent of the transaction price[4]) to the MP rule (where appraisal value is pegged to the winning bid). We demonstrate that under plausible real-world conditions, the specter of appraisal can play a value-enhancing role, rendering a categorical MP rule suboptimal. Our analysis also suggests conditions where the added welfare benefits of conventional appraisal are likely to be modest. These conditions include scenarios where management appears strongly biased *against* a sale, where the merger is subject to strong super-majority voting conditions for approval, and where the sales process involves robust participation by a large number of potential buyers.

Our results bear on several legal and policy debates related to appraisal. Most immediately, they shed skeptical light on the assertion—popular among the MP rule's proponents—that deal price is the best measure of fair value because it is forged in the "crucible of the market," thereby warranting deference from courts. While this argument is facially seductive, our analysis reveals it to be problematically circular: As our model demonstrates, the "market" is itself an endogenous byproduct of its own legal and regulatory environment (including fair-value criteria). Change that environment, and participants' expectations shift; shift expectations, and market prices move too.

More pragmatically, we view our results as both consistent with and instructive for emerging appraisal jurisprudence. As previously noted, the Delaware Supreme Court has recently resisted imposing a categorical rule mandating deal price deference in all arm's length sales. Rather, it opined that deal price should receive substantial deference only in those contexts where it is reasonable and consistent with accepted financial principles.[5] Our results support this contextualized approach, but they also inform it prospectively. By delivering insights about the contexts where merger price deference is—and is not—consistent with economic value creation, our analysis contributes to the body of accepted financial principles that may assist courts in considering whether and when to defer to merger price.

Additionally, our results have several empirical implications, shedding interpretive light on well-known regularities and motivating others. For example, our

[4] Discounted Cash Flow (DCF) is the dominant practice today, but all other methods that are independent of merger price also qualify, such as the comparable-companies approach. See Allen (2002).

[5] DFC Global Corp. v. Muirfield Value Partners, 172 A.3d 346, 347-8 (Del. 2017); Dell Inc. v. Magnetar, 177 A.3d 1, 5-6 (Del. 2017).

model predicts (consistent with prior efforts[6]) that under a conventional appraisal rule, (1) shareholders seek appraisal only for deals offering relatively low premiums, and (2) fair-value assessments will tend (in equilibrium) to skew *above* the deal price. Although proponents of a categorical MP rule often point to the upwards skew of appraisal awards as evidence of institutional dysfunction, our analysis suggests otherwise: the upward skew we predict is an artifact of rational, strategic decision making. (When target shareholders expect the appraisal valuation to be *lower* than the merger consideration, they will simply decline to seek appraisal.)[7] Our model similarly provides a useful framework for assessing the effects of "shocks" to appraisal law against broader welfare concerns (Boone et al. 2017; Callahan et al. 2017).

Finally, our analysis helps expose fundamental interactions between appraisal and other structural devices. For example, a popular deal structure for public-company targets in Delaware—and one where appraisal is typically available—involves a negotiated tender offer followed by an involuntary squeeze-out merger of non-tendering shareholders. Such two-step deals historically required at least 90-percent of target's shareholders to tender into the first stage.[8] In 2013, however, Delaware amended its statutes to allow an alternative "medium-form" merger, in which first step need secure only a 50-percent threshold before an accelerated squeeze out can commence.[9] A central result of our analysis (Proposition 6) is that deal price deference is more likely to be justified when the merger is conditioned on a strong super-majority approval of shareholders. This insight suggests that courts might similarly condition their appraisal approach on the strength of the shareholder mandate: for instance, traditional two-step deals requiring 90 percent support could receive the MP rule, while "medium-form" deals requiring only 50 percent would fall under more conventional approaches (such as DCF).

Three caveats to our core argument warrant elaboration before proceeding. First, a standard criticism of standard valuation approaches (such as DCF) is that they are prone to inaccurate calibration by judges who are not financially sophisticated.[10] Our analysis easily accommodates such possibilities. And, virtually all our arguments remain intact *even when* appraisal proceedings are subject to judicial estimation errors, so long as courts remain approximately unbiased overall. Indeed, much of the reserve-price benefit of appraisal inures to shareholders by enhancing buyers' willingness to pay higher premiums *ex ante*, so as to win affirmative votes and avoid appraisal. In the presence of prospective judicial error, rational buyers and sellers will simply replace a *known* appraisal value with its *expected* value. But so long as measurement errors remain more or less unbiased *ex ante*, substituting expected values will have trivial effects, and bidding and dissenting behaviors would remain largely unchanged.[11]

[6] See, e.g., Jiang et al. (2016); Korsmo and Myers (2015 and 2016).

[7] In fact, one would expect a similar upward skew *regardless* of whether the appraised value is set too high, too low, or just right by objective criteria. Our argument is also robust to adjustments that strip out buyer-specific "synergies" from fair value, per DGCL §262(h). See part IIC, *infra.*

[8] See DGCL §253. Falling short of 90% in the first stage may substantially slow or potentially kill a deal.

[9] See DGCL §251(h).

[10] See, e.g., Subramanian (2017).

[11] Of course, even when standard valuation approaches are approximately unbiased, they may be still costly to administer. In many cases, those costs are likely worth bearing given the significant monetary stakes

Second, our analysis largely follows convention within corporate law scholarship, assessing appraisal against a normative goal of maximizing the expected payoff of target shareholders as a whole. Although this assumption is natural within corporate law (and virtually compelled by both legal doctrine and jurisdictional competition), it is not the only plausible candidate. Other welfare objectives are also plausible (such as maximizing the sum of the target's and winning buyer's collective payoffs, or the payoff society in general, including non-shareholder constituents of both buyers and sellers). Even under such alternatives, however, most of our core observations remain qualitatively intact (See Section IV).

Third, although the threat of appraisal often tends to benefit target shareholders *as a whole*, its benefits need not be evenly distributed. In some equilibria of our model, appraisal risk increases expected payoffs of all shareholders on a pro rata basis. In other equilibria, however, payoffs can become skewed, dividing target shareholders into two groups: (1) those who seek (the more lucrative) appraisal remedy; and (2) those who must remain part of the majority that votes to support the deal (disqualifying them from appraisal). The egalitarian outcome tends to arise when shareholders cannot easily coordinate over who falls into group (1) versus group (2). With increased concentration of ownership among few sophisticated investors, however, non-egalitarian equilibria grow increasingly plausible.[12]

The remainder of this paper proceeds as follows. Section I presents a brief overview of related scholarship. Section II lays out the fundamental framework we study, combining auction design, shareholder governance, agency costs, and appraisal. Section III derives equilibria of the model for various appraisal rules. We show that voting and appraisal can interact in significant ways, with appraisal plausibly inducing strategic voting among shareholders. We also derive our central result that the MP rule is usually undesirable for target shareholders. Section IV considers a variety of extensions to our core model, including characterizing an optimal valuation measure for fair value. There we show that while the MP rule might, under the right circumstances, be one of many other optimal regimes, those circumstances appear to be relatively contingent. The last section concludes. All the proofs are in the Appendix.

I. Related Scholarship

This section briefly reviews three relevant lines of scholarship: (1) auction design, (2) shareholder voting, and (3) appraisal rights. As far as we know, ours is the first paper to interrelate all three dimensions. While scholarship on auction design is vast, its application to merger transactions is less extensive. Fishman (1988) shows why a buyer may be better off with a high, "preemptive" bid when information acquisition is costly,

involved. However, when the benefit of appraisal grow small (e.g., because of a robust auction process), administrative costs become prohibitive. Consequently, our analysis explicitly considers and characterizes conditions under which appraisal's welfare contribution is likely to be modest.

[12] Even here, however, retail investors still fare better than under the MP rule, though the optimal valuation measure tends to be larger too (resulting in even less egalitarian outcomes). *See* Section III(A), *infra*.

since such a preemptive bid can credibly signal to other bidders that the bidder has a high valuation for the target. Cramton and Schwartz (1991) analyze two important auction frameworks: private independent values or pure common values among the bidders. They argue that the perceived legal requirement on the target company to run an auction is better suited for the latter scenario than the former, because under a purely common value setting, the target can be sold to any buyer without any efficiency loss. Bulow and Klemperer (1996) demonstrate why a target company will be better off running (1) an auction with no reserve price but with one more bidder than (2) an auction with reserve price but with one less bidder. They demonstrate the importance of inducing more bidder participation. Che and Lewis (2007) examine the role of break-up fees and lock-ups in takeover contests and show that, when bidding is costly, break-up fees are generally more desirable because lock-ups tend to favor one bidder at the expense of another.

There also is a small number of academic studies that examine the effect of shareholder voting on corporate decision-making. Harris and Raviv (1988) examine different types of voting rules in the context where an incumbent and a rival compete over control. They argue that one-share-one-vote regime may be optimal because the rule does not create a bias in favor of either the incumbent or the rival. Stulz (1988) analyzes the effect of managerial control over voting rights on the probability and the size of a possible tender offer. He shows that, in the context where shareholders attach different valuations over the company, as the manager controls more voting rights, the probability of a tender offer falls but the tender offer premium rises. Bhattacharya (1997) examines shareholder voting issues in a proxy contest, where a dissent has to bear a cost to communicate its type ("good" or "bad") to or "lobby" the pivotal shareholder. The paper shows that as the communication cost falls, more proxy fights will ensue when the loss from electing a "bad" dissident is larger than the gain from choosing a "good" dissident. Recently, Becht, Polo, and Rossi (2016) examine the value of shareholder voting by looking at the effect on price from UK's imposition of mandatory shareholder vote in certain types of transactions. The paper finds that the shareholders generally gain from the imposition of mandatory shareholder voting.

A third line of relevant scholarship, developed principally by legal academics, deals with appraisal specifically. Fischel (1983) posits informally that appraisal risk provide *ex ante* benefits to target shareholders in a manner similar to that analyzed here. However, he ultimately endorses a valuation measure that our model shows to be strictly sub-optimal. He similarly does not account for other institutional factors that may substitute for appraisal (such as shareholder approval). In a related vein, Kanda and Levmore (1985) review the various theories associated with the appraisal remedy and argue that the appraisal remedy can be thought of as an additional check against agency problems. This idea also plays an important role here, too, as we explore how managerial incentives in selling the company can diverge from shareholders'. Thompson (1995) emphasizes the important role played by the appraisal remedy in giving minority shareholders an exit right. This is because, without appraisal, the majority can indefinitely retain the minority investment in an enterprise. Hermalin and Schwartz (1996) consider appraisal valuation in a setting where a majority shareholder can make an investment after a freeze-out and argues that the minority shareholders should be given

pre-investment value of the firm. Hamermesh and Wachter (2005) show how the existing Delaware case law has produced uncertainty in the concept of "fair value" especially when attempting to estimate the present value of future cash flows. Korsmo and Myers (2015 and 2016) document the recent rise of appraisal litigation and argue that appraisal plays a salutary role in mergers and acquisitions by playing proxies for deals that may hurt target shareholders.

While the empirical literature on appraisal remains relatively thin, several recent papers have shed additional light on the issue.[13] Jiang, Li, Mei, and Thomas (2016) present an empirical investigation of appraisal remedy and show that appraisal is more likely to be exercised when there is a perception of conflicts-of-interest and when the premium offered is low. The latter result, in particular, is consistent with our theoretical findings. Boone et al. (2017) and Callahan et al. (2017) both borrow the framework we develop here to investigate empirically whether more dissenter-friendly appraisal remedies enhanced shareholder welfare. Both find—consistent with our theoretical predictions—that target shareholders tend to receive higher premia as the strength of the appraisal remedy increases.

II. The Setup

We analyze the sale of a corporate entity ("target") involving three groups of risk-neutral players: target shareholders, an agent (or "manager"), and a group of potential buyers. Our game has four periods ($t \in \{0,1,2,3\}$) with no time discounting. At $t = 0$, dissenters' rights are fixed by law. At $t = 1$, bidders privately observe their respective valuations of the target and participate in an auction run by the manager. At $t = 2$, target shareholders can vote whether to accept the winning bid presented by the manager. Should a sufficient fraction vote in favor, the transaction closes, all shareholders relinquish their shares, and the assenting shareholders receive *pro rata* portions of the winning bid.[14] At $t = 3$, dissenting shareholders choose between (a) similarly accepting *pro rata* portions of the bid, and (b) obtaining court-appraised value.

We start with a description of the players. The target has a single class of fully-distributed stock, held by a countably large, diffuse group of $2T + 1$ shareholders ($T \in \mathbb{N}$ and $T \gg 0$), each owning a single share of the company. For expositional convenience, we will frequently describe the shareholder population as a continuum with mass 1, each holding a $d\gamma \approx \frac{1}{2T+1}$ fractional share of the company. That said, all equilibria described below derive from the discrete version of the model. Each shareholder places a different valuation on the firm as a going concern, indexed by her "type" $\gamma \in [\underline{v}, \overline{v}]$ with $0 \leq \underline{v} \leq \overline{v} < \infty$. Different valuations may be due to myriad factors, such as distinct tax positions,

[13] In an earlier study, using takeover data from 1975 through 1991 and comparing appraisal-eligible and appraisal-ineligible cases, Mahoney and Weinstein (1999) found little evidence that appraisal eligibility predicted different premia.

[14] Since dissenters *must* relinquish their shares, holdouts (Grossman and Hart 1980) are not as problematic in our model. We assume a single-step transaction for cash, but the assumption is easily relaxed.

liquidity preferences, non-convergent beliefs (with wealth constraints), and so forth.[15] Shareholder type γ values her ownership stake at $\gamma \cdot d\gamma$, and the entire firm at γ. Shareholder types are distributed according to a commonly-known cumulative distribution function $H(\gamma)$: $[\underline{v}, \overline{v}] \rightarrow [0,1]$ and associated density function $h(\gamma) > 0$ for all $\gamma \in [\underline{v}, \overline{v}]$.

Shareholders' heterogeneous valuations naturally cause disagreement about the relative attractiveness of bids. We distinguish between three shareholder types. First is the *marginal* shareholder with type $\gamma = \underline{v}$. The marginal shareholder effectively determines trading price, since her value reflects the lowest asking price for the stock in the absence of a merger.[17] Second is the *representative* shareholder with type $\gamma = \mu \in (\underline{v}, \overline{v})$, equal to the mean valuation among all target shareholders:

$$\mu = E(\gamma) \equiv \int_{\underline{v}}^{\overline{v}} \gamma \, dH(\gamma) \tag{1}$$

Third is the *pivotal shareholder,* denoted by $\gamma = \rho \in (\underline{v}, \overline{v})$, who holds the swing vote in approving any deal. The pivot is determined by threshold of shareholder approval needed to consummate the merger, which we denote by $\alpha \in [½, 1]$. Given a winning bid of b, all shareholders with $\gamma \leq b$ would prefer selling over maintaining the status quo, while shareholders with $\gamma > b$ do not. If shareholders vote sincerely, obtaining shareholder approval requires offering a sufficiently high price b such that $H(b) \geq \alpha$. Thus, with sincere voting, the pivotal shareholder is characterized by $\rho = H^{-1}(\alpha)$.

Turning to the agent, we assume that the manager[20] runs the auction among $N \geq 1$ bidders. For ease of exposition, we assume that the English, ascending-bid auction is chosen, where the bid starts at 0 and increases until only one bidder remains. Two important aspects about the manager deserve special mention. First, we assume the manager lacks the ability to set and commit to a reserve price. Second, the manager's private payoff may diverge from that of shareholders.[22] We assume that the manager

[15] The assumption of differential shareholder maps most closely into that of Stulz (1988), where differences in tax position among incumbent shareholders generates heterogeneous willingness to accept. In that setting, the shareholder with the lowest reservation value ($\gamma = \underline{v}$) sets market price. Shareholder heterogeneity can also occur when investors hold divergent beliefs about the company's value. See Brunnermeier et al. (2014). There, however, shareholder heterogeneity may not persist in equilibrium in the absence of wealth or liquidity constraints, since the shareholder with the largest valuation should be willing to purchase the shares from other shareholders. See Miller (1977) and Harrison and Kreps (1978). In such a case, the shareholder base collapses to a special case of our framework involves identically-valuing shareholders so that $\underline{v} = \overline{v}$. Our model nests this possibility too.

[17] Consistent with Stultz (1988), other investors with a lower valuation are assumed to have sold their stock at the market price, leaving the marginal shareholder as the price setter.

[20] Our model bundles together a variety of individual actors into the "manager" role, including corporate officers and directors as well as other actors who assist them with auction design—such as financial and legal advisers.

[22] See, e.g., RBC Capital Markets v. Jervis, 129 A.3d 816 (Del. 2015) (financial advisor manipulated board into accepting a proposed deal for which it had buy-side financing prospects); Smith v. Van Gorkom 488 A.2d 858 (Del. 1985) (retiring CEO sold the target too cheaply and with inadequate diligence).

maximizes $\Pi_m = \Pi_s + Pr(sale) \cdot M$, where Π_s denotes the expected payoff of the shareholders and $M \in \mathbb{R}$ denotes the manager's private gain or loss from the sale.[23] When $M > 0$, the manager realizes a private benefit from sale, making her "too eager" to sell. When $M < 0$, the manager enjoys a net private benefit of control under the status quo, making her "too reluctant" to sell. In the special case of $M = 0$, the manager's incentives are perfectly aligned with shareholders' interests.[24] We assume that M is commonly known by all players. The fact that the manager also cares about her private benefits from sale implies that, notwithstanding the inability to set a formal reserve price, she can refuse to sell when doing so makes her worse off. Her divergent interest, therefore, imposes a *de facto* reserve price (r_m), below which she will credibly refuse to sell. (As we will see shortly, r_m depends crucially on M.)

Finally, we assume that $N \geq 1$ bidders have been recruited to participate in the auction. We let N be exogenous for now, reserving for an extension the possibility of recruiting bidders. Each bidder $i \in \{1, \dots, N\}$ costlessly observes its private valuation of v_i. Our baseline analysis considers an independent private values (IPV) auction,[25] where v_i is independently and identically distributed on support $[0, \infty)$ according to a commonly-known cumulative distribution function $F(v)$, with continuously differentiable density function of $f(v) > 0 \ \forall v \in [0, \infty)$. We also make the standard regularity assumption that $\frac{1-F(v)}{f(v)}$ is monotone non-increasing in v. For purposes of exposition, we let $v_{(j)}$ be the j'th order statistic, such that $v_{(1)}$ as the lowest realization and $v_{(N)}$ as the highest realization of bidder valuations.

In terms of the timing of the game, at $t = 0$, the court (the "law") fixes and publicly announces the appraisal rule. For now we focus on two canonical appraisal rules. Under the market price (MP) rule, the court equates "fair value" (ϕ) with the winning bid ($\phi = b$). Under the conventional rule, the fair value is based on some other metric and the court's independent valuation (e.g., $\phi = \mu$). At $t = 1$, N bidders privately observe their valuations and participate in the auction. At $t = 2$, if the winning bid (b) is chosen and presented by the manager, shareholders get to vote whether to accept. When at least α fraction of the shareholders vote in favor, the merger gets consummated and the assenting shareholders receive the winning bid. At $t = 3$, conditional on there being a

[23] Note that the manager's objective function can be interpreted as one where the manager maximizes a convex combination of shareholder welfare and private benefits from sale (reflected by M). To avoid circularity, we omit from Π_s any components of shareholder payoff due to appraisal remedies.

[24] Although it is often intuitive to assume managers categorically have net private benefits of control under the *status quo* ($M < 0$), the opposite can easily hold in our framework. A variety of golden-parachute or post-merger employment guarantee can skew manager's incentives towards sale. Also, because our definition of "manager" amalgamates the interests of officers, directors, financial and legal advisers, providers of finance, etc. under a single banner, a pro-sale skew becomes particularly unsurprising.

[25] In the extension section, we discuss how our analysis extends to both common value (CV) and correlated private-value (CPV) auction settings. If the buyer's (independent) value stems from synergies, the buyer gets to realize that by only obtaining control of the seller. We assume, for simplicity, that the buyer must acquire 100% of the target. Also, we will assume away the possibility of reselling the target to a later buyer.

sale, those who voted against the merger may exercise their appraisal remedy so as to receive the fair value (ϕ) determined by the court.

III. Equilibrium Analysis

Because our game involves a sequential extensive form game with privately informed players, Perfect Bayesian Equilibrium (PBE) is an appropriate solution concept. It is important to remain mindful that both appraisal and shareholder approval provide potential checks on price adequacy, and they interact with each other. To better understand this interaction, we first analyze appraisal rights and voting rights in isolation. We present four distinct cases, as shown in Table 1.

	No SH Approval Right	*SH Approval Required*
No SH Appraisal Right	A	B
SH May Seek Appraisal	C	D

Table 1: Combinations of Shareholder Approval and Appraisal Rights

Our analysis works through each combination progressively: (A) a" benchmark" case where neither shareholder appraisal nor approval are possible; (B) the case where shareholders vote whether to approve a merger, but no appraisal is allowed; (C) the case where individual shareholders can seek appraisal but no shareholder vote is allowed; and finally (D) the case where both shareholder approval and shareholder appraisal are available and interact with one another. In working through cases (C) and (D), we also assess different valuation metrics for determining fair value, comparing the MP rule to the conventional rule that does not use the merger price as an input.

A. Benchmark Case: Pure Auction with No Shareholder Voting and No Appraisal Remedy

Consider first the benchmark case where shareholders have no voice whatsoever. As stated above, $N \geq 1$ bidders participate in the auction and the manager runs an English auction: a commonly observed bid opens at 0, and continuously rises until the earliest moment where a single bidder remains active.[26] As is well-known in the literature, for each player i, the dominant strategy is to stay in the auction until the bid surpasses his valuation v_i.[27] The probability of a sale for $N \geq 1$ number of bidders and reserve price $r \geq 0$, therefore, is $\Pr\{Sale|N,r\} = 1 - F(r)^N$. As is well-known in the literature, given the shareholders' mean valuation of μ, the optimal reserve price $r^* \in [\underline{v}, \overline{v}]$, which maximizes the shareholders' expected payoff, is given by:

$$r^* = \mu + \frac{1 - F(r^*)}{f(r^*)} \tag{2}$$

[26] Known as the revenue equivalence principle, in an independent and private value setting, all four standard auctions—first-bid, second-bid, English, and Dutch auctions—produce the same (expected) revenue for the seller. See Krishna (2002) at 29-36.

[27] See Krishna (2002) and Ausubel and Cramton (2004).

With $f(\cdot) < \infty$ and $F(\overline{v}) = 1$, we have $r^* \in (\mu, \overline{v})$.[28] If shareholders could commit to a reserve price, r^* would be a logical choice. However, it is the *manager* who designs the auction process, an insight that renders the following result:

Proposition 1. *When neither shareholder approval nor appraisal are available, there is a unique equilibrium in which the firm is sold to the highest bid of $b \geq r_m^*$, where:*

$$r_m^* \equiv max\{\mu - M, 0\} \tag{3}$$

Whenever $M > -\frac{1-F(r^*)}{f(r^*)}$, we get $r_m^* < r^*$. When $M > 0$, the sales price can even result in a shareholder payoff falling short of the status quo ($b < \mu$).

If shareholders have no voice in monitoring the sale process, the manager will attempt to maximize her own payoff of Π_m without any constraint. Furthermore, the manager's inability to commit to a reserve price implies that she will accept any highest bid b that promises more than her payoff under the status quo ($b + M \geq \mu$), thereby effectively setting the *de facto* reserve price of $\mu - M$. Whenever the manager's private benefit from the status quo is not "too extreme" (i.e., when $M > -\frac{1-F(r^*)}{f(r^*)}$), the manager becomes too eager to sell and the *de facto* reserve price falls below the shareholders' optimal reserve price ($r_m^* < r^*$). When M grows sufficiently large and positive, the manager becomes willing to accept a bid that could result in an expected loss to the target shareholders. For instance, in the case of a single bidder and $M > \mu$, we get $r_m^* = 0$, resulting in a payoff for target shareholders of 0.[29] On the opposite end, when the manager enjoys a "large" private benefit from the status quo ($M < -\frac{1-F(r^*)}{f(r^*)}$), she becomes too reluctant to sell, and the *de facto* reserve price grows too large ($r_m^* > r^*$).

B. Shareholder Approval but No Appraisal Remedy

Now consider the case where $\alpha \in [1/2\,, 1]$ fraction of the shareholders must vote to approve a sale, but appraisal rights remain unavailable. As is well known in the political science literature, voting models with many players generically have multiple (infinitely many) equilibria. The usual culprit is indifference: for any posited equilibrium where a clear winner emerges, no single player's vote is "pivotal" in determining the

[28] The condition above is closely related to the monopoly pricing problem, where the seller sets price by balancing the chance of no sale against the hope of a higher winning bid (Bulow and Klemperer (1996)). Note further that the optimal reserve price is clearly in excess of the pre-announcement market price ($\underline{v}$), a measure endorsed by Fischel (1983).

[29] When the manager must expend effort to recruit bidders, the single-bidder case may be a real possibility, since one bidder is enough for the manager can secure a sale and obtain her private payoff. In the extension section, we discuss the possibility where N will be endogenously determined through the manager's effort. Similarly, we have assumed that the buyers do not engage in costly search to find the target company. If they need to do so, such costly search will reduce their ex ante expected return. Providing incentive to the target manager to recruit potential buyers can function as a (partial) substitute for buyers' costly search. See Gilson and Schwartz (2017).

outcome. To deal with the multiplicity issue, we deploy a version of standard equilibrium refinement that involves the *elimination of weakly dominated strategies*. (Duggan (2003) and Patty et. al. (2009)) The refinement disallows any posited equilibrium strategy $\hat{\sigma}_\gamma$ for any player γ if there exists an alternative strategy $\tilde{\sigma}_\gamma \neq \hat{\sigma}_\gamma$ that fares at least as well for player γ across *every* possible permutation of opponents' strategy profiles $\boldsymbol{\sigma}_{-\gamma} \in \boldsymbol{\Sigma}^{-\gamma}$, and does strictly better for player γ in at least one such permutation. In our framework, the weak dominance refinement is sufficient to generate sincere voting and a unique equilibrium. We will refer to the set of limiting-case equilibria that remain after removal of weakly dominated strategies as *weakly undominated equilibria*.

With the refinement, the preferences of the pivotal shareholder with valuation $\gamma = \rho$ begin to loom large. In particular, if the highest bid falls short of ρ, then the pivotal shareholder and all those of types $\gamma \geq \rho$ will disfavor it, vote against the merger, and no transaction will be consummated. Only bids that offer at least the pivotal shareholder's value become feasible. Effectively, shareholder approval introduces a second *de facto* reserve price at the pivotal shareholder's value.

Proposition 2. *When a fraction of shareholders of at least* $\alpha \in [½, 1]$ *is required to approve a merger but appraisal is unavailable, there is a unique set of outcome-equivalent, weakly undominated equilibria that are revenue equivalent to an auction with reserve price of* $max\{r_m^*, \rho\}$. *Shareholders vote sincerely, and they approve the merger when the winning bid is at least equal to* ρ. *The equilibrium payoff exceeds the shareholders' payoff in the absence of approval when* $r_m^* < \rho \leq r^*$.

Note that the requirement of a shareholder vote can be useful to the manager, too. By imposing an external minimum threshold price for the deal to go through, the specter of a shareholder vote confers on the manager commitment power to walk away should the highest bidder offer any less. As is shown in the proof of Proposition 1, if the manger could commit to a reserve price, she would set it equal to $r_m^{**} = \frac{1-F(r_m^{**})}{f(r_m^{**})} + \mu - M > r_m^*$. For a manger who cannot commit to a reserve price, having the de facto reserve price of $max\{r_m^*, \rho\}$ through the voting requirement can enhance her private return.

Although the discussion above treats ρ as fixed (e.g., enshrined in the merger statute), there may be ample room for tailoring. For example, the target's charter might contain a "supermajority" provision, conditioning fundamental changes on a supererogatory level of shareholder support, even as high as $\rho = r^* \Leftrightarrow \alpha = H(r^*)$, so that even without appraisal, shareholder approval alone may be able to support an optimal reserve price. Alternatively, the deal may explicitly condition closing on receiving a supermajority of "yes" votes or a no more than a maximal threshold of dissenters. Such provisions are not uncommon in negotiated acquisitions.[31] Here, it would be up to the manager and bidders to negotiate a supermajority structure described above;

[31] A related provision, known as a "blow" provision, has a similar effect, allowing the buyer to back out if more than a critical mass of shareholders seek appraisal. We discuss this in the extension section.

consequently, the manager's commitment constraint and skewed incentives would conspire to defeat such efforts without external compulsion.

C. Appraisal with No Shareholder Approval

Now consider the case where an appraisal option is available to shareholders, but there is no shareholder vote.[32] Without a shareholder vote, once the winning bid (b) is determined, all shareholders may choose between (1) accepting the winning bid (b) and (2) petitioning the court to determine the "fair value" (ϕ) of their shares. We assume this is paid by the winning buyer in lieu of the merger consideration to all petitioning, dissenting shareholders, while the non-petitioning shareholders are cashed out on the terms of the merger. We compare two regimes: (1) the MP rule, where, ϕ is pegged at the winning bid (b); and (2) a "conventional" rule, where, ϕ is fixed through a procedure that remains independent of the winning bid.

1. The Merger Price (MP) Rule

First consider the effects of the MP rule, where the fair value is pegged to the merger price: $\phi = v_{(N-1)}|v_{(N-1)} \geq r_m$. Now, consider a shareholder of type γ who is choosing between accepting the terms of a merger at a winning bid of $b = v_{(N-1)}|v_{(N-1)} \geq r_m$ or seeking a judicial appraisal and receiving the same amount $\phi = b$. This shareholder gains nothing from seeking appraisal over simply accepting the merger terms. From the winning buyer's perspective, there is no difference either. The strategic role that the MP rule serves in equilibrium is significant: because the MP rule provides no "outside option" to the merger price, it imposes no meaningful constraint on (de facto) reserve prices, thereby leaving such choices completely up to the manager. The resulting strategic landscape, then, is identical to the benchmark case (from subsection A), where there was neither appraisal nor approval.

Remark 1. *When appraisal is available at an amount pegged to the merger price, and there is no shareholder approval, the unique equilibrium is identical to the case where neither appraisal nor approval were permitted (Proposition 1).*

Although Remark 1 is in many ways obvious, we state it formally to underscore its pertinence to ongoing policy debates about deal price deference. At least when viewed in isolation, the MP rule is tantamount to *eliminating the appraisal remedy altogether,* and with it whatever shareholder value enhancements that it might deliver under a variety of valuation measures.

2. The "Conventional" Rule

Under the "Conventional" rule, the appraisal is pegged at some fixed value ϕ, chosen by the court. We assume for the moment that ϕ is fixed *ex ante*, at a level

[32] This scenario has practical relevance, as in the context of acquisitions of controlled firms or squeeze outs. See, e.g., DGCL §253 ("short-form" merger). The company can also have non-voting stock.

commonly observed by all players. Beyond requiring that it be untethered to the winning bid price b, for now we remain agnostic about *exactly* how ϕ is computed. That said, several possibilities suggest themselves. First, fair value might be pegged against the valuation of the representative shareholder: $\phi = \mu$. Such a measure has some intuitive appeal, since it represents an aggregated measure of the target's going-concern value over its incumbent owners. Alternatively, ϕ might be pegged to the optimal reserve price that the representative shareholder would set in an auction: $\phi = r^* = \mu + \frac{1-F(r^*)}{f(r^*)}$. This measure also has some intuitive appeal, since, under this rule, the expected return of the shareholders will be maximized. When choosing between the winning bid of b and judicially determined fair value of ϕ, regardless of the shareholder's type, it is clear that she would favor appraisal in all cases where the merger price falls below ϕ, and favors the merger terms otherwise. Similar to shareholder voting, the Conventional rule imposes a *de facto* reserve price ϕ.

Proposition 3. *When appraisal is available under the Conventional rule in the amount ϕ but there is no shareholder approval, there is a unique set of outcome-equivalent equilibria generating revenue equivalent to an auction with reserve price of $max\{r_m^*, \phi\}$. Expected shareholder welfare is (weakly) greater than the benchmark case (Proposition 1) for all $\phi \in [0, r^*]$, (weakly) greater than the status quo value of μ whenever $\phi \in (\mu, r^*]$, and (weakly) greater than under shareholder approval alone (Proposition 2) when $\phi \in (\min\{\rho, r^*\}, \max\{\rho, r^*\})$.*

The result above is very close to that of Proposition 2, other than the replacement of ρ with ϕ. The size of the shareholders' payoff in the appraisal-only case depends critically on how ϕ is set. If the court is free (and sufficiently competent) to choose ϕ near r^*, target shareholders likely fare (weakly) better under appraisal only than when limited to approval-only. On the other hand, if the voting rule induces the pivotal voter ρ to be near r^*, the opposite can hold, and an approval-only regime can dominate.

3. Comparison of MP and Conventional Rules

With the above results in hand, we can offer a preliminary assessment of how the MP rule stacks up against the Conventional rule within the appraisal-only regime. It is easy to see that this doctrinal battle is not very flattering for the MP rule. The MP rule leaves target shareholders in the same position as if they had no dissenters' rights. This is problematic in at least two respects. First, the threat of a conventional appraisal proceeding can help the manager to commit credibly to a higher reserve price. Second, conventional appraisal tends to help target shareholders when the manager has limited ability or incentive to commit to a value-maximizing reserve price. In fact, *even if a court were incompetent* and unable to discern with sufficient accuracy any of those plausible measures, it could still (at least weakly) enhance target shareholder welfare beyond what the MP rule promises simply by fixing fair value at a trivially low level (such as $1). Doing so would not hurt, and it night plausibly help in the case where management's incentives are too skewed towards selling.

The MP rule tends to suppress target shareholder value for all N, but its comparative disadvantage attenuates as competition grows. To see this, consider Figure 2, which plots the expected welfare of target shareholders (Π_s), as a function of the reserve price (horizontal axis) and the number of bidders (depth axis). The figure assumes that the uniform distribution on the unit interval governs both the target shareholders' and the buyers' valuations. For the sake of comparison, suppose $M \geq \mu$, so that the de facto reserve price is equal to zero. Note the hyperplane cutting through the figure at the point where the reserve price is equal to $r = 0.75$, which is the optimal reserve price (r^*). The MP rule effectively reduces the reserve price to zero, represented by the far-left wall of the graph. The MP rule represents a significant hit to target shareholder payoffs when the number of bidders is small (e.g., in the low single digits). However, as the number of bidders increases (moving up on the depth axis), the penalty visited by the MP rule shrinks substantially. While the MP rule is still worse than the optimal reserve price, its advantage dissipates considerably with more competition.[36]

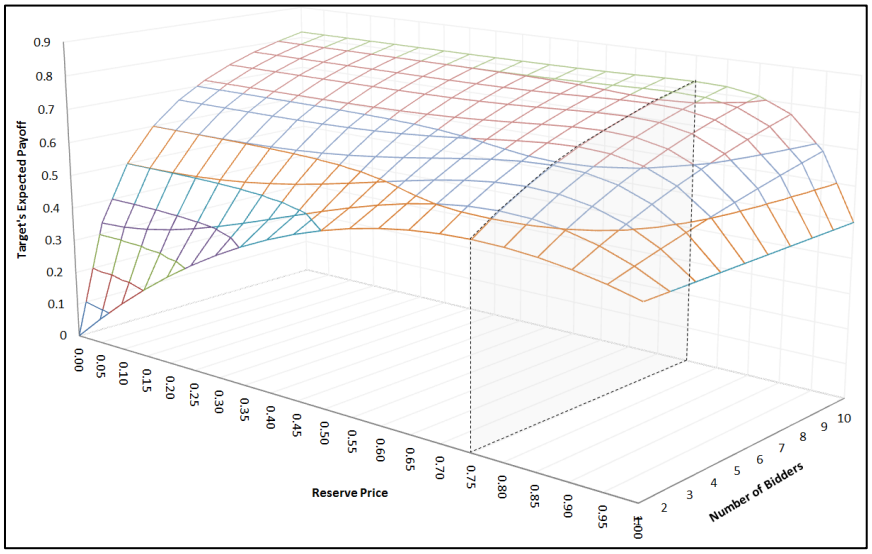

Figure 2: Target Shareholders' Expected Payoff as a Function of Reserve Price (Horizontal Axis) & Number of Bidders (Depth Axis).

The intuition above extends to the general case too; it is straightforward to show that the marginal value of increasing r attenuates in N, and that this marginal return eventually tends to zero as N grows arbitrarily large.[37] Thus, in the case of appraisal with no voting, the adoption of the MP rule over plausible alternatives—while always suboptimal for shareholders in a *qualitative* sense—visits a somewhat limited discount on them in a *quantitative* sense with competitive bidding. We revisit this point below.

D. Shareholder Approval Combined with Appraisal

[36] Bulow and Klemperer (1996) demonstrates this effect more generally, showing that the value of a reserve price can be swamped by the value of adding another bidder.
[37] For a formal proof of this claim, see Lemma 1A in the Appendix.

Finally, consider the hybrid case where both shareholder approval is required and appraisal is available to the dissenters. This is the most interesting and complex case, since we must consider not only the effects of both options in isolation, but also their interaction. Such interaction is, in fact, a virtual certainty under current law, due, at least, to two key aspects. First, under most states' appraisal statutes (including Delaware's), target shareholders are ineligible to seek appraisal if they previously voted in favor of the merger.[38] In the context of our model, this implies that shareholders must vote against the merger to be eligible. Second, shareholders who vote against the merger receive a true option—they may select whether to take the merger consideration or seek appraisal. Thus, voting in favor of a merger extinguishes real option for the target shareholder, while voting against preserves it.[39]

Accordingly, now each shareholder's strategy consists of two elements: (1) determining whether to vote in favor of or against the merger; and (2) conditional on having voted against the approved merger, deciding whether to accept its terms or seek appraisal. Unlike the previous cases, the combination of these factors can support equilibria with strategic (insincere) voting by shareholders who prefer the merger yet nonetheless demur to preserve eligibility to seek appraisal. We once again start with the "Merger Price" rule, and then move on to several plausible "Conventional" rules.

1. The Merger Price (MP) Rule

As seen in the previous subsection, with the MP rule, the appraised fair value floats mechanically up and down with the merger price, so that, for any given reserve price $r \geq 0$, we have $\phi = v_{(N-1)} | v_{(N-1)} \geq r$. Just as before, no shareholder ever gains from seeking appraisal, and the game devolves into the pure approval rights case.

Proposition 4. *When shareholders must approve the winning bid and appraisal is based on the MP rule, there is a unique set of outcome-equivalent, weakly undominated equilibria identical to that characterized in Proposition 2.*

With the MP rule, the dissenting shareholders would receive the same consideration as the assenters. As in Proposition 2, the winning bidder needs to, at minimum, induce the pivotal voter to vote in favor of the bid and the de facto reserve price is given by $\max\{r_m^*, \rho\}$. When $\max\{r_m^*, \rho\} < \mu$, it is possible for a buyer to acquire the company for less than its incumbent shareholders value it. The MP rule negates the effect of the appraisal remedy, leaving shareholder voting as the exclusive source of reserve price protection for target shareholders.

2. The Conventional Approach

[38] See, e.g., DGCL §262(a) (appraisal available to shareholder who "neither voted in favor of the merger or consolidation nor consented thereto in writing").

[39] See, e.g., DGCL §262(e) (allowing dissenting shareholder, who previously notified the corporation its intent to exercise the appraisal remedy, to withdraw and accept the merger consideration within 60 days of the completion of the merger).

Now consider the Conventional appraisal remedy, in which appraisal is pegged to a fixed value ϕ untethered to the merger price. Appraisal introduces yet another form of reserve price on the bidding process, since a bid that falls below appraisal value is sure to elicit negative votes. Whether this added reserve affects equilibrium behavior turns on the size of ϕ relative to alternative reserve prices. We analyze two critical orderings:

i. Case A: $\phi < \max\{r_m^*, \rho\}$

When $\phi < \max\{r_m^*, \rho\}$, appraisal has no effect, since it is dominated by alternative reserve prices. Any approved deal would have a bid in excess of $\max\{r_m^*, \rho\}$, and no shareholder would seek appraisal, as reflected in the following Lemma.

Lemma 5A. *If shareholder approval and appraisal are both available and the appraisal remedy is pegged at* $\phi < \max\{r_m^*, \rho\}$*, there is a unique set of outcome-equivalent, weakly undominated equilibria identical to that characterized in Proposition 2.*

ii. Case B: $\phi \geq \max\{r_m^*, \rho\}$

Now consider the more interesting case where appraisal exceeds the reserve price established by voting and managerial bargaining: $\phi \geq \max\{r_m^*, \rho\}$. A successful merger can now result in an appraised fair value that the pivotal shareholder would find attractive relative to the status quo. Whether (and how) the shareholder responds to this incentive depends on the value of the winning bid b. On one end of the spectrum, if the winning bid b were even higher than the appraisal value ($b \geq \phi \geq \max\{r_m^*, \rho\}$), no shareholder would ever favor appraisal, since the terms of the merger dominate the anticipated appraisal award. Here, the appraisal option does no added work, and the merger is supported by a strong majority (all voting sincerely). The merger always succeeds.

Suppose, in contrast, the winning bid is low: $b < \max\{r_m^*, \rho\} \leq \phi$. Here, the pivotal shareholder (the ρ-type) has divided interests: appraisal looks attractive, while the merger price is unattractive. The pivotal shareholder's most preferred option would be to see merger consummated over her "no" vote and then to seek appraisal; and if that route were unavailing, she would want the merger to fail. Either way, she finds it weakly dominant to cast her vote against the merger. Similar reasoning also applies to all shareholders with valuation weakly exceeding the pivotal shareholder's. Consequently, the merger always fails. The reasoning from the two sub-cases above are reflected in the following Lemma.

Lemma 5B.1. *When* $\max\{r_m^*, \rho\} \leq \phi \leq b$*, the unique weakly undominated equilibrium of the voting continuation game prescribes sincere voting and approval of the merger. When* $b < \max\{r_m^*, \rho\} \leq \phi$*, the unique weakly undominated equilibrium prescribes sincere voting and rejection of the merger.*

The most interesting case is when the winning bid resides in the Goldiloxian middle: $\max\{r_m^*, \rho\} \leq b < \phi$. Here, the type ρ shareholder is sure to find the winning bid attractive, but she finds appraisal even more lucrative. The pivotal shareholder would

most prefer that the transaction be consummated and then to seek appraisal. However, her next most preferred strategy would be for the merger to be approved and to receive the winning bid. Her least preferred strategy is the outright rejection of the merger. And herein lies the rub: for the type ρ shareholder, in order to retain eligibility for her most preferred outcome (appraisal), she must vote insincerely for her least preferred outcome (rejection). Such an appraisal-preserving negative vote would be acceptable to the pivotal shareholder *if* she could count on other shareholders to carry the requisite majority (α) to override her vote.

But alas, all shareholders with valuations on the interval $[\underline{v}, b]$ are performing the same strategic calculus, hoping that others will vote to support the merger so that they can reject the merger and seek appraisal. For this group of shareholders, weak dominance no longer does any work in refining possible equilibria. A collective action problem ensues, and much depends on whether merger-supporting shareholders coordinate on a voting equilibrium that determines who can seek appraisal, and who must "take one for the team" to approve the deal. It should therefore not be surprising that there are multiple equilibria in this case. Confining attention to pure-strategy equilibria,[40] two distinct classes of equilibria emerge in this case.

Lemma 5B.2. *When* $\max\{r_m^*, \rho\} \leq b < \phi$*, there are two classes of weakly undominated equilibria in pure strategies of the voting continuation game:*

(A) In the first ("uncoordinated") equilibrium, a coalition of shareholder types $Z_1 \subset [\underline{v}, b]$ *consisting of strictly less that the needed majority α vote in favor of the merger, with all others voting against. All voting against seek appraisal. The merger never succeeds.*

(B) In the second ("coordinated") equilibrium, a coalition of shareholder types $Z_2 \subseteq [\underline{v}, b]$ *comprising an exact α-fraction of shareholders vote in favor of the merger, and all others vote against. All those voting against seek appraisal. The merger always succeeds.*

As Lemma 5B.2 illustrates, the appraisal interacts non-trivially with shareholders' voting incentives. When the merger price is attractive to the requisite majority of shareholders but the anticipated appraisal value is even more lucrative, equilibrium turns on whether shareholders on the interval $\gamma \in [0, b]$ can cobble together a "coalition of the willing" to support the merger. If they cannot coordinate, the bid is rejected even though a majority of shareholders would have preferred it. When they succeed in coordinating, the merger wins by a hair's breadth, and the "no" voters (many of whom have voted insincerely) seek appraisal, extracting a higher expected price.

[40] Although we cannot generically exclude mixed strategy equilibria, weak dominance excludes all equilibria in which players on $(b, \overline{v}]$ vote for the merger with positive probability. Moreover, of the remaining shareholders on $[\underline{v}, b]$, there exist no symmetric mixed strategy equilibria prescribing vote in favor of the merger with probability $p \in (0,1)$. See the Appendix for details.

Note that in the second, "coordinated" equilibrium, the buyers' bidding strategy must adapt as well. The buyer must anticipate the possibility of having to pay two different prices: (i) the bid amount to the assenters (the $\alpha = H(\rho)$ fraction voting in favor and receiving b), and (ii) a premium price to the dissenters (the $(1-\alpha)$ fraction voting against and receiving appraisal value of $\phi > b$). The buyer's total outlay therefore may exceed of the winning bid b and be equal to $\max\{b, \alpha b + (1-\alpha)\phi\}$. Consequently, buyers must prepare to adjust their bidding behavior to account for the implicit "tax" they pay. Analysis of the foregoing lemmas yields the following central result.

Proposition 5. *When shareholders vote on the winning bid and Conventional appraisal is available, equilibrium turns on the relative sizes of ρ and ϕ:*

(A) When $\phi < \max\{r_m^*, \rho\}$, all weakly undominated equilibria are identical to Propositions 2 and 5 and we have the de facto reserve price equal to $max\{r_m^*, \rho\}$. Winning bids are approved and no dissenters seek appraisal.

(B) When $\phi \geq max\{r_m^*, \rho\}$, there are two classes of weakly undominated equilibria in pure strategies.

(1) In the first, winning bids are always at least ϕ, and are approved without dissent with no shareholders seeking appraisal. The equilibria are identical to those in Propositions 4, and revenue equivalent to an auction with reserve price ϕ.

(2) *In the second, winning bids are always at least ρ. If the winning bid exceeds ϕ, it is approved without dissent with no shareholders seeking appraisal. Otherwise, the winning bid is approved by a bare α-fraction of target shareholders, with the remainder seeking appraisal. The equilibrium is revenue equivalent to an auction with reserve price of* $\max\{\alpha\rho + (1-\alpha)\phi, r_m^*\}$.

Although Proposition 5 is somewhat involved, its intuitive content is simple. The appraisal rule "matters" only if it is not overshadowed by alternative types of deal price protection. When $\phi < max\{r_m^*, \rho\}$, the appraisal option is insufficiently potent to move the needle, since the required vote on the merger already ensures a *de facto* reserve price of at least ρ. Here, there is no difference between the MP Rule and the Conventional rule: both are effectively moot.

Once the appraisal value grows sufficiently large ($\phi \geq max\{r_m^*, \rho\}$), equilibrium behavior changes significantly, pushing the *de facto* reserve price above ρ. How far above turns on which class of equilibrium emerges. In the "uncoordinated" equilibrium (Lemma 5B.1), shareholders' collective action problem causes them to reject any bid below ϕ, which then becomes the de facto reserve price for the auction. When the "coordinated" equilibrium (Lemma 5B.2) obtains, voting and appraisal interact. Those seeking appraisal must rely on sufficiently many affirmative voters to approve the deal and make appraisal possible, and all shareholders voting to approve the merger

effectively become pivotal. Bidders' thus expect to pay a two-part price consisting of the winning bid (to an α-fraction of shareholders) and the appraisal value (to the remaining $1-\alpha$). The end result is to replicate the expected payoffs of an ascending auction with *de facto* reserve price equal to $\max\{\alpha\rho + (1-\alpha)\phi, r_m^*\}$.

Several aspects of the equilibria described in Proposition 5 warrant attention. First, our model predicts appraisal will be far from ubiquitous. When it occurs in equilibrium, it will be systematically pursued only in those circumstances where (a) the anticipated appraisal award exceeds the pivotal voter's valuation ($\phi > \rho$); (b) the "coordinated" equilibrium obtains; and (c) the winning bid lies somewhere between ρ and ϕ. In no other cases are appraisal proceedings an equilibrium phenomenon.

Second, in those cases where appraisal does occur in equilibrium, *fair-value assessments will systematically exceed the announced merger price.* This ordering holds regardless of whether ϕ is set "too high" or "too low" as measured against some benchmark. It is simply a byproduct of equilibrium behavior: strategic litigants will tend to pursue appraisal only when they expect it to be more attractive than the winning bid. Consequently, one should be skeptical about the argument that the appraisal system is "broken" because appraisal awards typically exceed the merger price.[41] Such evidence may well demonstrate that parties are acting rationally; but it is not necessarily a symptom of institutional dysfunction warranting the broad embrace of the MP rule.[42]

Finally, for any fixed $\phi \leq r^*$, the expected revenue from the "coordinated" equilibrium (B)(1) is strictly less than its counterpart in the "non-coordinated" equilibrium (B)(2). Target shareholders' collective ability to coordinate can ultimately hurt them in the aggregate, by allowing bidders to rely on a judicially-mediated price discrimination, paying a lucrative appraisal value to dissenters but a more modest bid to "yes" voters. In fact, the recent emergence of sophisticated hedge funds (such as Merion Capital and Elliot Associates) pursuing appraisal might signify a transition of sorts from uncoordinated to the coordinated equilibria, in which uncoordinated shareholders must more frequently carry the burden of merger approval while strategic and coordinated investors reap the greater benefits of dissenting and seeking appraisal. That said, this seemingly inequitable outcome does not in itself justify the adoption of the MP rule. Indeed, a comparison of Propositions 4 and 5 reveals that regardless of the equilibrium that emerges, target shareholders, *as a class*, are at least weakly better off under a Conventional approach than the MP rule (at least so long as $\phi \leq r^*$).[43]

[41] See, e.g., Bainbridge (2012) and Hamermesh and Wachter (2005).
[42] Accord Bomba et al. (2014) (asserting similar conclusions from several practitioners' standpoints).
[43] In fact, as we show in the next section, the optimal judicial response to the increased frequency of the coordinated equilibrium may be (ironically) to *increase* the award even further.

IV. Applications and Extensions

With the equilibrium analysis in hand, we can now discuss a variety of applications and extensions. We first highlight the question of designing an "optimal" appraisal rule, followed by a brief discussion of several other possible extensions.

A. Optimal Appraisal Policy in (Possibly) Error-Prone Courts

Our first application relates to a core motivation in this paper: assessing the conditions under which the MP rule would be an "optimal" judicial policy. For present purposes, we define "optimality" as the appraisal approach that maximizes the expected return that the target shareholders as a whole realize in equilibrium.[44]

Proposition 6. *The optimal appraisal rule is characterized as follows:*

(A) If $max\{r_m^*, \rho\} > r^*$*, then the optimal appraisal rule is not unique and includes both the merger price and any fixed* $\phi \leq max\{\rho, r_m^*\}$*;*

(B) If $max\{r_m^*, \rho\} \leq r^*$ *, then the optimal appraisal rule is unique and characterized as follows:*

(1) *If the "uncoordinated" equilibrium in Proposition 6(B)(1) obtains, then the optimal rule fixes* $\phi^* = r^*$*;*

(2) *If the "coordinated" equilibrium in Proposition 6(B)(2) obtains, then the optimal rule fixes* $\phi^* = \left(\frac{r^* - \alpha\rho}{1-\alpha}\right) > r^*$.

Proposition 6 shows, among other things, that the MP rule *may* be one of many optimal rules in certain circumstances, but only when either (a) the pivotal shareholder's valuation, $\rho = H^{-1}(\alpha)$, exceeds the optimal reserve price for the target shareholders (r^*); or (b) the manager enjoys a private benefit of control under the *status quo* that is "large" (so that $r_m^* > r^*$). When either of these conditions hold, then any fair value rule that renders appraisal unattractive (including the MP rule) can be optimal.

This observation naturally tees up the question of when the conditions stated in Proposition 6 would obtain and make the MP rule defensible. One possible circumstance involves mergers that require a strong *supermajority* of shareholders to approve. To take one example, recall that two-step mergers in Delaware traditionally required at least 90% of the target shareholders to tender their shares into a first step tender offer before the squeeze-out step was permitted.[45] This is tantamount to setting $\rho = H^{-1}(0.9)$. Proposition 6 tells us that when a merger requires supermajority assent such as this, either

[44] Although this is a natural definition for current purposes, the language of the statute does not compel the court to adopt it. We thus discuss other possible welfare objectives below.

[45] See DGCL §253. As noted above, the Delaware legislature recently promulgated DGCL §251(h), allowing a buyer to commence with a squeeze out contingent on a bare 50% of the target shareholders tendering into the first step tender offer.

by compulsion or by the pursuit of a certain deal structure, the pivotal voter's preference may provide sufficient pricing protection, and it would be optimal to relax appraisal standards, possibly by adopting the MP rule. More succinctly, the MP rule is potentially defensible in the presence of strong super-majority mandates to approve the merger.

Another possibility arises when the target's management team enjoys relatively large net private benefits of control from the status quo (i.e., $M < -\frac{1-F(r^*)}{f(r^*)}$). Here, entrenchment will cause the manager to set the reserve far too high from the perspective of target shareholders. In such circumstances, neither appraisal nor shareholder voting offers valuable pricing protection. The MP rule becomes more defensible when directors' and officers' had strong ex ante entrenchment incentives.[46]

Significantly, Proposition 6's insights are largely robust to environments where valuation assessments are inexact or prone to considerable statistical noise (possibly due to judicial error). This observation is not insignificant, since the perceived imprecision of conventional appraisal approaches such as DCF as applied by (putatively) inexpert judges is often cited as a key rationale for using the merger price (Subramanian (2017)). Our framework easily accommodates such conditions. To see why, suppose in attempting to implement a target valuation amount ϕ, the court is prone to err, so that the realized appraisal value in litigation is $\phi' = \phi + \varepsilon$, where ε represents a noise term independent of ϕ, with zero mean and finite variance. In this setting, both bidders and shareholders would predicate their strategies *not* on ϕ', but on the expected value[47] of ϕ'. But because $E(\phi') = \phi$, all the results from Proposition 6 continue to go through, and the error term would have little net effect (so long as the noise component remains unobservable until the case is decided[48]).

Finally, it is important to note that this brief discussion presumes an "optimal" appraisal rule to be one that maximizes target shareholders' ex ante expected net payoff. While this assumption is natural within the context of corporate law, some judges could harbor objectives that coincide with a broader measure of social welfare.[49] If we were to adapt our analysis to embrace such desiderata, it would have implications for several of our results. As is well-known, the optimal reserve price in an English auction is analytically equivalent to a monopolist's profit maximizing pricing condition, balancing the chance of failing to make an efficient sale on the margin against the reward of a

[46] Even with an entrenchment incentive, if the manager is subject to the Revlon duty to maximize the return for the shareholders, the manager may be unable to realize his/her private benefits of control. In such a setting, the conventional appraisal remedy may boost the return for the target shareholders.

[47] While this argument presumes risk neutrality, little changes with risk aversion, since the judicial noise affects *both* the buyer and dissenters, giving them a strong incentive to settle at close to expected value.

[48] A caveat worth noting is that the option-like nature of appraisal could introduce *bias* if information about judicial error is observed *after* signing but *before* dissenters must commit to seek appraisal. Dissenters would then effectively own a call option over valuation risk, and it would be appropriate to *reduce* the optimal appraisal award by the value of that option. (A similar adjustment would be warranted for a variety of other sources of bias.) Net of such adjustment, however, the core results in Proposition 6 remain intact.

[49] One plausible reading of the statute, for example, might constrain a judge to award no more than the status quo value of the target as measured by the representative agent, μ. As we discuss below, such a measure may also be an optimal one more generally in the case of common-value auctions.

higher price on the infra-margin. Were we to incorporate both buyers' and sellers' expected welfare, it would be optimal to set the reserve price at $r^* = \mu$, so that the company always ends up in the hands of the highest valuing player. The key steps of our earlier analysis would go forward, but with the caveat of $r^* = \mu$. That alteration, in turn, would expand the circumstances under which the MP rule could be optimal by slackening the conditions in which $\rho \geq r^* \equiv \mu$.

B. Other Extensions

There are several other extensions and applications of the basic analysis that are worth consideration. The first concerns equilibrium selection. Proposition 5 is silent on the question of equilibrium selection, but this issue is important, especially for the buyers with valuations between $\alpha\rho + (1-\alpha)\phi$ and ϕ, who may need to decide whether to participate in the auction. One possible way of confronting this issue is to identify situations where shareholders are likely to be able to solve their coordination problem. A plausible predictor is ownership concentration in the firm. Though we have assumed that the target stock is completely dispersed, this may be overly simplistic. For many publicly traded companies, there are a relatively small number of institutional investors holding large blocks of the outstanding stock. In that setting, the bare majority equilibrium (Lemma 5B.2(B)) may be easier support, compared to a setting where the shares are completely dispersed and the shareholders are wholly unorganized.

Multiple equilibria may also pose challenges for the judges. To deal with this uncertainty, the appraisal remedy might be contingent on the realized equilibrium: the judge may be able to "learn" which equilibrium is in play by observing the voting outcomes before deciding on the appraisal rule. For instance, when the merger agreement sets the threshold relatively high ($\rho \geq r^*$), there may be little need for an appraisal remedy that pushes that de facto reserve price even higher (with $\phi > \rho$). In such cases, it might be better for the court to adopt the Market Price (MP) rule, or put evidentiary weight on the merger price as corroborative of fair value, so as to eliminate possible distortion that can be caused by the Conventional rule. This intuition suggests that judges might apply the MP rule when the appraisal follows a two-step merger involving a 90% squeeze out condition than a 50% condition. Alternatively, if the shareholder vote was a close call, the judge could infer a coordinated equilibrium has obtained and set the fair value under the appraisal at $\phi^* = \frac{r^* - \alpha\rho}{1-\alpha}$. If the merger proposal receives a robust approval, on the other hand, the judge could infer that it was an uncoordinated equilibrium, and set the appraisal value at $\phi^* = r^*$. An attractive feature about such a contingent appraisal system is that it awards more compensation to the dissenting shareholders when the merger seems more controversial.

We might additionally extend the analysis by introducing judges as strategic players. Recall from above, the judicial attraction to the MP rule is due (in part) to the technical demands that alternative approaches (such as DCF) place on law-trained judges. Siding with the merger price effectively reduces a judge's personal cost of generating an appraisal value. And, even if the judge is aware that pre-committing to the deal price ex ante undermines the reserve-price effect of appraisal, it can be tempting to side with it *ex*

post (after the *ex ante* effects have become irrelevant). Moreover, in multi-judge jurisdictions, each judge's aversion to technical difficulties in valuation can be compounded further by a free riding incentive—since the identity of the presiding judge is not determined until after a deal is closed and appraisal actions are filed. Such considerations might cause individual judges to be *too* smitten with the MP rule, even as they understand its bid-dampening effects.

Our analysis also allows us to examine a variety of different contractual mechanisms that respond to appraisal risk. From the buyer's perspective, appraisal can introduce transactional uncertainty. Not surprisingly, some buyers try to reduce or eliminate such surprises through a variety of contractual terms. One often-observed contractual clause, known as a "blow" provision, allows the buyer to walk away from the deal if a sufficient fraction of target shareholders exercise the appraisal remedy. Especially if the bare majority equilibrium is anticipated (Proposition 5), the buyer may have a strong incentive to adopt such a condition so as to protect itself against a cascade of dissenters. A blow provision that is set at, say 20%, will allow the winning bidder to avoid such an outcome. At the same time, blow provisions also implicitly condition the deal on a supermajority vote to consummate the deal (80% in this case). This side effect may ultimately benefit target shareholders, since it requires the buyer to increase its bid to be attractive to a super-majority of shareholders. Other contractual mechanisms include "drag-along" provisions, which require shareholders to vote in favor of the merger under certain conditions and lose appraisal, and "naked no vote" fees, which require the target to pay the buyer a termination fee in the event of a negative vote by shareholders. All else held constant, our model predicts that they would tend to dilute bids and shareholder welfare.

Another potential line of extension concerns the auction environment. We have focused on the tractable setting of independent, private valuations (IPV) among the bidders. This assumption may be too restrictive, but we can extend our analysis to allow correlation among bidder valuations: correlated private valuations (CPV) or common valuations (CV). Doing so affects our analysis in several ways. Most notably, the optimal reserve price in the IPV setting is relatively aggressive, but with correlated values, the optimal reserve price will decline towards μ.[50] Even in such settings, the traditional appraisal remedy can still play an important role when the pivotal shareholder's valuation falls below the average valuation: $\rho < \mu$. By setting $\phi = \mu$, the court can avoid the possibility of the target firm being sold to a buyer with inefficiently low valuation ($\rho \leq v_i < \mu$). On the other hand, when the pivotal shareholder's valuation is higher than the average valuation, $\rho \geq \mu$, with strong correlation in bidder valuations, the MP rule might be an efficient response.

[50] If the seller does not have any information that could affect the buyer's valuations, as the number of bidders grows, it may no longer be optimal to set a reserve price above the seller's valuation (μ in our model). See Levin & Smith (1996) at 1279 (showing that the optimal reserve price in a correlated values auction converges to the seller's true value as the number of bidders grows arbitrarily large); and Krishna (2002) at 121—124 (failure of the "exclusion principle").

Finally, we have assumed throughout that the number of bidders (N) is exogenous and fixed a priori. We can enrich the model by allowing the manger to recruit new bidders. Such an extension would not only add richness into the auction model but also into the principal-agent setup. For example, suppose it costs the manager $k > 0$ to attract a new bidder. In that setting, an optimal appraisal rule may depend on the number of bidders in an interesting way: $\phi^* = \phi(N)$ where $\phi'(N) \leq 0$. Under this structure, the manager would have an incentive to heat up the bidding through recruiting buyers, and the court would reward her efforts by progressively reducing the effective reserve price. One version of this mechanism would be for the court to (progressively) revert to the MP rule as the number of buyers increases. One possibility to adopt a cut-off rule, where the court reverts to the MP rule when the number of bidders is sufficiently high: $\phi^* = MP$ when $N \geq N^* > 1$ and $\phi^* = \phi$ otherwise. Another possibility is to assign differential weights on the merger price and the traditional appraisal valuation, depending on the number of bidders and let the weight on the MP rule grow as the number of bidders gets larger: $\phi^* = q(N) \cdot MP + (1 - q(N)) \cdot \phi$ where $q(N) \in [0,1]$ and $q'(N) \geq 0$. Such contingent rules could function as an incentive device for the deal team.

Conclusion

Post-merger appraisal rights have garnered significant attention recently, due in part to rise of "appraisal arbitrage" by several sophisticated investors. Responding to concerns about speculative petition activity, as well as the challenge of divining independent valuations, Delaware courts have grown increasingly amenable to using the merger price itself as an important lodestar for determining "fair value." A principal objective of this paper has been to evaluate whether and how adoption of the MP rule would affect the ex ante structuring of the sales process itself. Our analytic framework (which combines auction design, agency costs, and shareholder voting) helps shed light on how anticipated appraisal rights and valuation protocols affect this process. We have demonstrated that appraisal is an important mechanism not only in protecting the dissenting shareholders' rights after the fact, but also in affecting their interests ex ante, by imposing a de facto price floor (reserve price) on bidding.

This analytic exercise delivers several insights about when the MP rule would (and would not) be desirable from an economic welfare perspective. Foremost, the MP rule tends overall to depress both acquisition prices and target shareholders' expected payoff compared to both the optimal appraisal rule and the conventional approach that sets the "fair value" independent of the merger price. That said, our analysis has also suggested specific conditions under which the MP rule may be close to optimal, such as when the deal is structurally dependent on super-majority shareholder approval, or when used as an incentive device to encourage a deal team to recruit a healthy number of interested buyers. These situations square reasonably well with what appear to be the several contours of doctrine as it is developing in the courts. That said, our analysis can help to inform and shed light on what circumstances would be needed to justify the adopting MP rule over more conventional valuation approaches.

Beyond these insights, our model helps explain why a healthy majority of litigated appraisal cases using conventional fair value measures result in valuation assessments exceeding the deal price, an equilibrium phenomenon predicted by our analysis and a simple artifact of rational, strategic behavior (not necessarily an institutional deficiency, as some have suggested). In addition, our analysis facilitates a better understanding of the strategic and efficiency implications of recent reforms allowing "medium-form" mergers, as well as various appraisal-related practices, such as blow provisions, drag-alongs, and "naked no-vote" fees.

Finally, the equilibrium framework developed here can be used to derive several concrete predictions and comparative statics, which in turn are amenable to empirical testing. Although far beyond the scope of our enterprise here, several authors have begun to employ our framework to assess a variety of "shocks" that altered the availability or profitability of pursuing the remedy (See, e.g., Callahan, Palia & Talley 2017; Boone, Broughman & Macias 2017). Progress in testing predictions from our analysis sheds light on a host of other interesting debates that surround post-acquisition appraisal.

Appendix: Proofs

Proof of Proposition 1. The proof closely follows the standard in auction theory. For more detail, see the treatments by Milgrom (1987) and Krishna (2002). Suppose the manager runs an English auction with a credible reserve price of $r \geq 0$ to maximize $\Pi_m = E(shareholder\ return) + Pr(sale) \cdot M$. For bidder i, let $y = max\{v_1, \dots, v_{i-1}, v_{i+1}, \dots, v_N\}$. The cumulative distribution and density functions of y are $G(y) = F(y)^{N-1}$ and $g(y) = (N-1)F(y)^{N-2}f(y)$. Conditional on $v \geq r$, a bidder expects to pay $m(v;r) = r \cdot G(r) + \int_r^v y \cdot g(y)dy$. The bidder's ex ante expected payment is:

$$E_v\{m(v;r,N)\} = \int_r^{\infty} \left(r \cdot G(r) + \int_r^v y \cdot g(y)dy \right) f(v)dv$$
$$= r \cdot G(r) \cdot [1 - F(r)] + \int_r^{\infty} [1 - F(y)] \cdot y \cdot g(y)dy$$

The total payoff for the seller is:

$$\Pi_s(\mu, r, N) = N \cdot E_v\{m(v;r,N)\} + F(r)^N \mu$$
$$= N \cdot \left(r \cdot G(r) \cdot [1 - F(r)] + \int_r^{\infty} [1 - F(y)] \cdot y \cdot g(y)dy \right) + F(r)^N \mu$$

When we maximize $\Pi_s(\mu, r, N)$ with respect to r, we get:

$$\Pi_s(v_0; r^*, N) = N \cdot \left[1 - (r - v_0) \frac{f(r)}{1 - F(r)} \right] (1 - F(r)) \cdot G(r)$$

From this, a simple but helpful lemma follows:

Lemma 1A: *As N increases beyond $N = 1$, the marginal value of setting the reserve price r decreases for all values of r. In the limit, as N grows arbitrarily large, the marginal value of adjusting r approaches zero.*

Proof of Lemma 1A: First, note that for all r and $N > 1$,

$$\frac{\partial^2 \Pi_s(v_0; r, N)}{\partial r \partial N} = \left[1 - (r - v_0)\frac{f(r)}{1 - F(r)}\right]\big(1 - F(r)\big) \cdot F(r)^{N-1}(1 + N \cdot \ln(F(r)) < 0$$

which is negative for $r < r^*$ and strictly positive for all $r > r^*$. A simple application of L'Hopital's rule to the above expression confirms that

$$\lim_{N \to \infty} \frac{\partial^2 \Pi_s(v_0; r, N)}{\partial r \partial N} = 0$$

Thus, as N grows arbitrarily large, the value of setting $r > 0$ is fully attenuated. ■

With a credible reserve price, given that the probability of sale is given by $1 - F(r)^N$, the manager's objective function can be written as:

$$\Pi_m(\mu, r, N, M) \equiv \Pi_s(\mu; r, N) + (1 - F(r)^N)M$$

Lemma 1B: *If the manager can credibly commit to a reserve price, when maximizing her expected return, she will set the reserve price at*

$$r_m^{**} = \frac{1 - F(r_m^{**})}{f(r_m^{**})} + \mu - M$$

Proof of Lemma 1B: When we take the derivative of the manager's objective function respect to r, after some simplifications, we get:

$$\frac{\partial \Pi_m(\mu, r, N, M)}{\partial r} = N\big(1 - F(r)\big)F(r)^{N-1} \cdot \left[1 - (r - \mu + M)\frac{f(r)}{1 - F(r)}\right]$$

Note that the second derivative is strictly negative for all interior r, so the interior root of the above equation must be a unique maximum. This unique maximum occurs at when the expression in the square brackets is zero, or:

$$r_m^{**} = \frac{1 - F(r_m^{**})}{f(r_m^{**})} + \mu - M$$

When $M = 0$, we get $r_m^{**} = r^*$. With the assumption that $\frac{1-F(v)}{f(v)}$ is monotone and non-increasing in v, we see that when $M > 0$, $r_m^{**} < r^*$ and when $M < 0$, $r_m^{**} > r^*$. Finally, as M increases (decreases), r_m^{**} decreases (increases). ■

Now, suppose that the manager cannot credibly commit to a reserve price. Suppose the highest bid from the auction is $b \geq 0$. If the manger were to sell the company, she realizes $b + M$. If she refuses to sell, her expected return is μ, which is the average valuation of the shareholders. She will agree to sell whenever $b + M \geq \mu$ or $b \geq \mu - M$. The de facto reserve price is given by $\max\{\mu - M, 0\}$. ■

Proof of Proposition 2: We start with the voting equilibrium. Recall that the continuous framework presented in the text approximates for the limiting case of a finite-voter game, $\Gamma(H, b, \alpha; T)$, in which there are $2T + 1$ shareholders ($T \in \mathbb{N}$ and $T \gg 0$), each holding a fractional ownership claim of $\frac{1}{2T+1} \approx d\gamma$. Suppose that each shareholder has valuation $\underline{v} = \gamma_0 < \gamma_1 < \cdots < \gamma_i < \cdots < \gamma_{2T+1} = \overline{v}$, and that the winning bid is $b \in [0, \infty)$. The merger requires a fraction $\alpha \in [1/2, 1)$ of affirmative votes. For any T, this is equivalent to requiring at least $T_\alpha \geq \alpha \cdot (2T + 1)$ affirmative votes. The payoff shareholder of type j gets from the status quo is $\gamma_j \cdot d\gamma$, while the payoff the shareholder gets from an accepted bid is $b \cdot d\gamma$.

There are multiple Nash equilibria in this game, including those where the bid wins (or loses) by more than one vote, so that no voter considers herself "pivotal." The weak dominance refinement restricts attention to those situations where a voter views herself as pivotal. For each player of type γ_j, weak dominance requires that the probability of voting in favor of the deal is zero whenever

$$\gamma_j \cdot d\gamma > b \cdot d\gamma \Leftrightarrow \gamma_j > b$$

Similarly, the probability of voting to approve the deal is 1 when $\gamma_j < b$. If $\gamma_i = b$, we assume, without loss of generality, the shareholder votes for the merger with probability 1. We define the pivotal voter as the shareholder who has valuation γ_j such that:

$$H(\gamma_{j-1}) < \alpha \leq H(\gamma_j) \Leftrightarrow \gamma_{j-1} < H^{-1}(\alpha) \leq \gamma_j$$

Let $\gamma^*(\alpha)$ denote the valuation of the pivotal voter. The reasoning above establishes.

> **Lemma 2A:** *There exists a unique weakly-undominated equilibrium of $\Gamma(H, b, \alpha; T)$ for all b and T. All shareholders for whom $\gamma_j \leq b$ vote in favor of the merger, while all those for whom $\gamma_j > b$ vote against. The merger is approved if and only if $b \geq \gamma^*(\alpha)$.*

Finally, consider the limiting behavior of $\Gamma(H, b, \alpha; T)$ as $T \to \infty$. Observe that $\lim_{T\to\infty} H(\gamma^*(\alpha)) = \alpha$, and thus $\lim_{T\to\infty} \gamma^*(\alpha) = \rho$. This immediately implies:

> **Lemma 2B:** *The limiting case equilibrium of* $\Gamma(H, b, \alpha; T)$ as $T \to \infty$ *is unique for all b. All shareholders for whom* $\gamma \leq b$ *vote in favor of the merger, while all those for whom* $\gamma > b$ *vote against. The merger is approved if and only if* $b \geq \rho \equiv H^{-1}(\alpha)$.

Turning to the manager's incentive, from Proposition 1, we had the de facto reserve price of $r_m^* = \max\{\mu - M, 0\}$. When $r_m^* \leq \rho$, the new de facto reserve price becomes ρ, whereas when $r_m^* > \rho$, the de facto reserve price stays at r_m^*. Since $\rho > 0$, the de facto reserve price is given by $\max\{r_m^*, \rho\}$. ■

Proof of Proposition 3. Suppose that the agent agrees to a merger at price b. With no shareholder voting, the strategy choice for each shareholder is to choose between taking the merger consideration b or seeking appraisal at a "fair value" equal to ϕ. If $b < \phi$, conditional on there being a merger, the dominant strategy is to seek appraisal for all shareholders. If $b > \phi$, the dominant strategy for all shareholders is to eschew appraisal and receive b. If $b = \phi$, all shareholders are indifferent between accepting the merger terms or seeking appraisal. As before, we assume that the shareholders accept b.

Given the shareholders' strategies, for each bidder, if she wins the auction with $b < \phi$, she must pay ϕ. When $b \geq \phi$, she pays b. Consequently, any bidder with valuation $v < \phi$ will immediately drop out, whereas if $v \geq \phi$, she will stay in the auction until the bid reaches her valuation. This is equivalent to an auction with the reserve price of ϕ.

The equilibrium de facto reserve price is given by $r = \max\{r_m^*, \phi\}$. If $v_{(N)} < r$, which occurs with probability $F(r)^N$, the firm will not be sold. On the other hand, if $v_{(N)} \geq r$, the firm will be sold at price equal to $\max\{r, v_{(N-1)}\}$. The ordering of shareholder welfare follows naturally from the definition of r^*. ■

Proof of Proposition 4. Suppose the winning bid is given by $b \geq 0$. Under the MP rule, the dissenting shareholders who exercise the appraisal remedy receive b. Therefore, they are indifferent between exercising and not exercising the appraisal remedy. Assuming, for simplicity, that they do not exercise the remedy, given that the shareholders get to vote on the merger, the equilibrium is identical to that in Proposition 2. ■

Proof of Lemma 5A. In addition to the condition $\phi < \max\{r_m^*, \rho\}$, suppose also that $r_m^* \geq \rho$. It is clear that the lowest bid that the manager would permit shareholders to vote exceeds ϕ. Consequently, any strategy involving appraisal is strictly dominated, leaving only the vote in question. Under weak dominance in voting (see Proposition 2), all shareholders with type $\gamma_i \leq b$ must vote in favor with probability 1, and all those with type $\gamma_i > b$ vote against. But because $b \geq r_m^* > \rho$, the it is clear that any bid satisfying the manager's reserve must garner a greater than an α share of votes.

Now suppose instead that $\rho > r_m^*$. Here, the manager will allow all bids exceeding r_m^* to go to a shareholder vote. Consider three cases.

Case 1: Consider a winning bid b such that $r_m^* \leq b < \phi < \rho$ and shareholder type $\gamma \in (\phi, \overline{v}]$. Define three scenarios: (1) merger takes place regardless of the shareholder's vote; (2) merger fails regardless of the shareholder's vote; and (3) the shareholder's vote is pivotal. In the first, the shareholder's dominant strategy is to vote against the merger and exercise appraisal. In the second, the shareholder is indifferent across different strategies. In the third, the shareholder's dominant strategy is to vote against the merger and seek appraisal. Hence, for all shareholders with $\gamma \in (\phi, \overline{v}]$, the weakly dominant strategy is to vote against the merger and seek appraisal. The merger fails.

Case 2: $\phi \leq b < \rho$. Since $b \geq \phi$, appraisal is a dominated strategy and can be excluded. Consider shareholder type $\gamma \in (b, \overline{v}]$. In scenario (1), the dominant strategy is to vote against and not exercise appraisal. In scenario (2), the shareholder is indifferent among different strategies. In scenario (3), the shareholder's dominant strategy is to vote against the merger. The shareholder's weakly dominant strategy, once again, is to vote against the merger and seek no appraisal in case the merger takes place. The bid does not get enough votes from the shareholders and the merger fails.

Case 3: $\phi < \rho \leq b$. Since $b \geq \phi$, seeking appraisal is again a dominated strategy. Consider shareholder type $\gamma \in [0, \rho]$. In scenario (1), the dominant strategy is for the shareholder to vote for the merger. In scenario (2), the shareholder is indifferent among different strategies. Finally, in scenario (3), the dominant strategy is to vote for the merger to receive $b \geq \gamma$. The weakly dominant strategy for the shareholder with type $\gamma \in [0, \rho]$ is to vote for the merger and the merger will succeed. All shareholders receive the consideration of b and no one exercises the appraisal remedy.

In sum, the auction is revenue equivalent to a simple auction with a de facto reserve price of $\max\{r_m^*, \rho\}$. The equilibrium is therefore identical to that in Proposition 2. ■

Proof of Lemma 5B.1. We consider the two cases in sequence.

Case 1: $\{r_m^*, \rho\} \leq \phi \leq b$. Since $\phi \leq b$, appraisal is a dominated strategy. Consider shareholder type $\gamma \in [0, b]$. In scenario (1), the dominant strategy is for the shareholder to vote for the merger. In scenario (2), the shareholder is indifferent among different strategies. In scenario (3), the dominant strategy is to vote for the merger. Hence, the weakly dominant strategy for the shareholder is to vote for the merger. Since $\rho \leq b$, merger succeeds and all shareholders receive the merger consideration.

Case 2: $b < \max\{r_m^*, \rho\} \leq \phi$. Suppose first that $r_m^* \geq \rho$. By construction, the manager credibly refuses any offer $b < r_m^*$, and thus there would never be a vote and the merger would never be consummated. Now suppose instead that $r_m^* < \rho$ Because $b < \phi$, all dissenting shareholders seek appraisal. Consider shareholder type $\gamma \in (b, 1]$. In scenario (1), the dominant strategy is for the shareholder to vote against the merger and seek appraisal. In scenario (2), the shareholder is indifferent across strategies. Finally, in scenario (3), the dominant strategy is to vote against the merger since $\gamma > b$. The weakly dominant strategy for the shareholders on $(b, 1]$ is to vote against the merger and seek appraisal remedy. For similar reasons, The merger will fail since $b < \rho$.■

Proof of Lemma 5B.2. Suppose $\max\{r_m^*, \rho\} \leq b < \phi$. Whenever a merger is conjectured certain to occur at b, all shareholders would prefer to vote against and seek appraisal. That outcome is clearly not attainable. Thus, any equilibria involving the approval of the merger can never have more than a bare majority in support, thereby making every affirmative vote pivotal. The key issue is how the "no" votes are allocated among the shareholders.

Consider shareholders with $\gamma > \phi$. For these shareholders, it is weakly dominant to vote against the merger, since they do not want the deal under any circumstances, and they would rather receive the appraisal if it does occur. A similar reasoning applies to shareholders with type $\gamma \in (b, \phi]$. They would most prefer to see the merger approved, but seek appraisal, which requires them to vote against. If that is not possible, their next best outcome is that the merger not approved, which also prescribes voting against. Their least preferred scenario is to vote for a merger. Thus, this group will vote against the merger as well. All shareholders with $\gamma > \mathrm{b}$ will vote sincerely against the merger.

Now consider the shareholders with $\gamma \leq b$, which includes $\gamma = \max\{r_m^*, \rho\}$. While all such shareholders would support the merger on its own terms, they would prefer to seek appraisal if they knew the deal would be approved. This creates a coordination problem. If there were an equilibrium where all of the shareholders voted for the merger, all would have a strict incentive to defect and vote against and seek appraisal. Hence, there cannot be an equilibrium involving approval unless votes in favor marshal an exact α fraction. There are infinite ways to marshal this vote, but in all of them, the relatively low-valuing shareholders (for whom $\gamma \leq b$) must coordinate on a way to ration their no votes so as to preserve the approval of the merger. In contrast, there are an arbitrarily large number of equilibria in which these low-valuing shareholders overwhelmingly vote no. Both types of equilibria are robust to the elimination of weakly dominated strategies. ■

It is also possible to show that there exists no mixed strategy equilibrium, where all shareholders vote in favor of the merger with a strictly positive, but less than one, probability, when $\max\{r_m^*, \rho\} \leq b < \phi$.

Proof of Proposition 5. Part (A) follows from Lemma 5A where ρ is the de facto reserve price. The manager imposes the de facto reserve price of $\max\{r_m^*, \rho\}$. Part (B)(1) follows from Lemmas 5B.1. With ϕ as the de facto reserve price in the uncoordinated equilibrium, the results are identical to those in Proposition 3.

Now, suppose $\phi \geq \rho$, and we are in a coordinated equilibrium. From Lemma 5B.2, without any reserve price, the winning bid must be at least equal to ρ. Furthermore, when the winning bid $b \in [\rho, \phi)$, the winner effectively pays $\alpha b + (1-\alpha)\phi$. That is, when $b \in [\rho, \phi)$, the effective payment is between $\alpha\rho + (1-\alpha)\phi$ and ϕ. Finally, when $b \geq \phi$, per Lemma 5B.2, all shareholders accept the bid and the winner pays b. Consider three cases. First, if $r_m^* < \alpha\rho + (1-\alpha)\phi$, given that the winning bid will never be less than ρ (and the effective payment – and thus *de facto* reserve price – is never less than $\alpha\rho + (1-\alpha)\phi$), the manager would never wish to push the reserve still upwards pricing

pressure and would thus set her own nominal reserve price no higher than ρ. Second, if $r_m^* \geq \phi$, the manager, per Proposition 3, maximizes the return by setting $r_m = r_m^*$. Third, if $\alpha\rho + (1-\alpha)\phi \leq r_m^* < \phi$, the manager would want the winning bidder to make an effective payment at least equal to r_m^*. This can be achieved by choosing r_m such that $\alpha r_m + (1-\alpha)\phi = r_m^*$ or $r_m = \frac{1}{\alpha}(r_m^* - (1-\alpha)\phi)$. ■

Proof of Proposition 6. Recall, from Part II.A, that $r^* > \mu$ and r_m^* may be larger or smaller than r^* depending on whether *M* is negative or positive. First, suppose $\rho > r^*$. Per Propositions 4 and 5, setting $\phi > \rho$ will (weakly) reduce the target shareholders' expected return. The optimal appraisal rule is to either use the MP rule or to set $\phi \leq \rho$.

Second, suppose $\rho \leq r^*$. Now, voting threshold is insufficiently high to maximize the target shareholders' return. The optimal appraisal rule depends on the type of equilibrium obtained and the manager's private incentive. Per Proposition 5(B)(1), in the uncoordinated equilibrium, ϕ becomes the de facto reserve price. The optimal appraisal rule, therefore, is to set $\phi = r^*$. Note that when $M > 0$, because $r_m^* < r^*$, setting $\phi = r^*$ will make the shareholders strictly better off in expectation.

In the coordinated equilibrium, per Proposition 5(B)(2), with $\rho = H^{-1}(\alpha)$, α fraction of the shareholders receive b while the remaining shareholders get ϕ. The expected payment by the winning bidder is $\alpha b + (1-\alpha)\phi$. The minimum winning bid is equal to ρ, in which case the buyer's expected payment is $\alpha\rho + (1-\alpha)\phi$. To create the de facto reserve price of r^*, the court needs to set ϕ such that $\alpha\rho + (1-\alpha)\phi = r^*$ or, equivalently, $\phi = \frac{r^* - \alpha\rho}{1-\alpha}$, subject to the constraint of $\frac{r^* - \alpha\rho}{1-\alpha} \leq 1$. The optimal appraisal rule, therefore, is: $\phi = \min\left\{1, \frac{r^* - \alpha\rho}{1-\alpha}\right\}$. ■

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